InClass_Chapter10_MomentOfInertia.pptx

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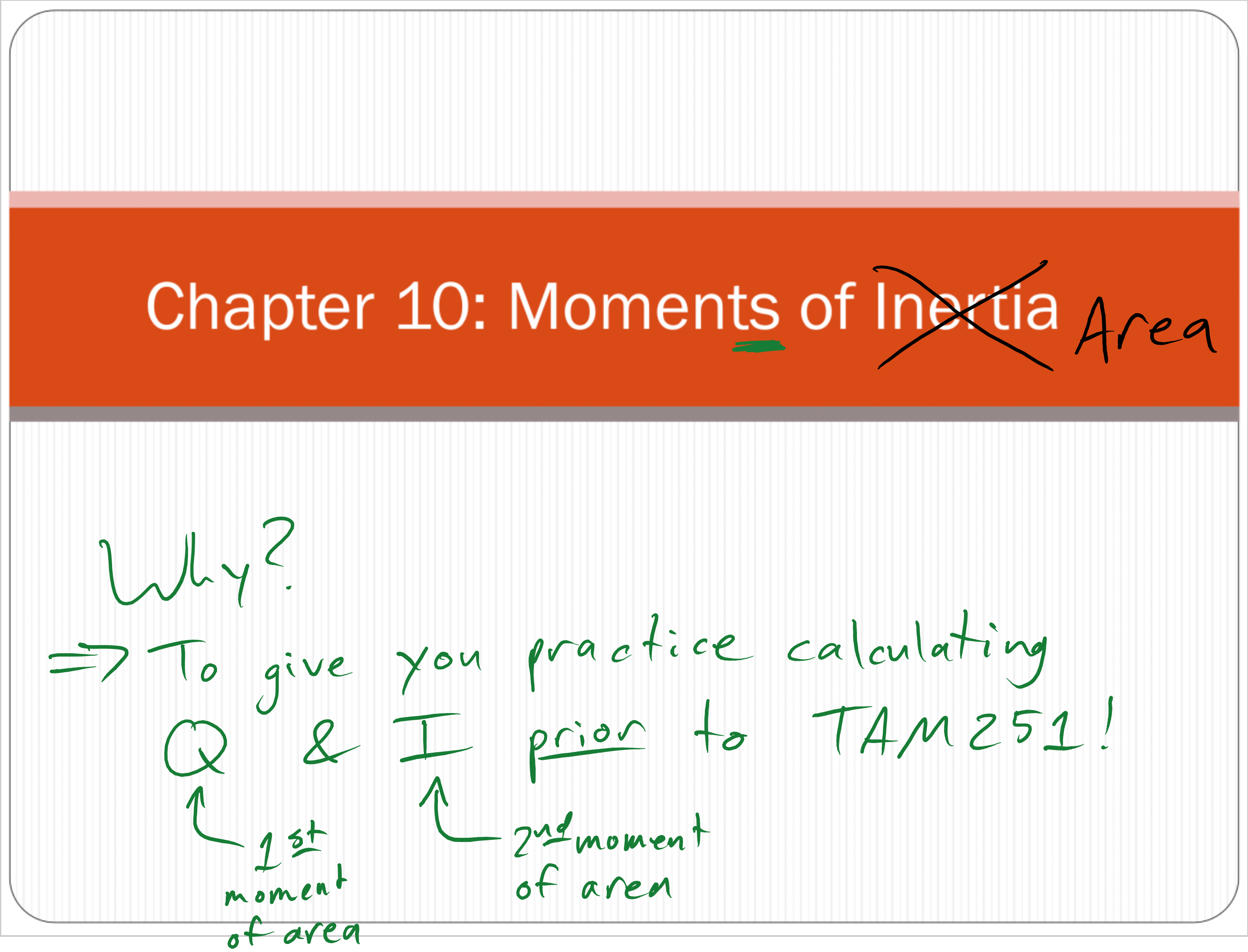

Chapter 10: Moments of ~~Inertia~~ Area

Why?

⇒ To give you practice calculating Q & I prior to TAM 251!

Q ← 1st moment of area

I ← 2nd moment of area

For TAM 251

Q ⇒ related to shear stress in a beam

I ⇒ related to a beam's resistance to bending

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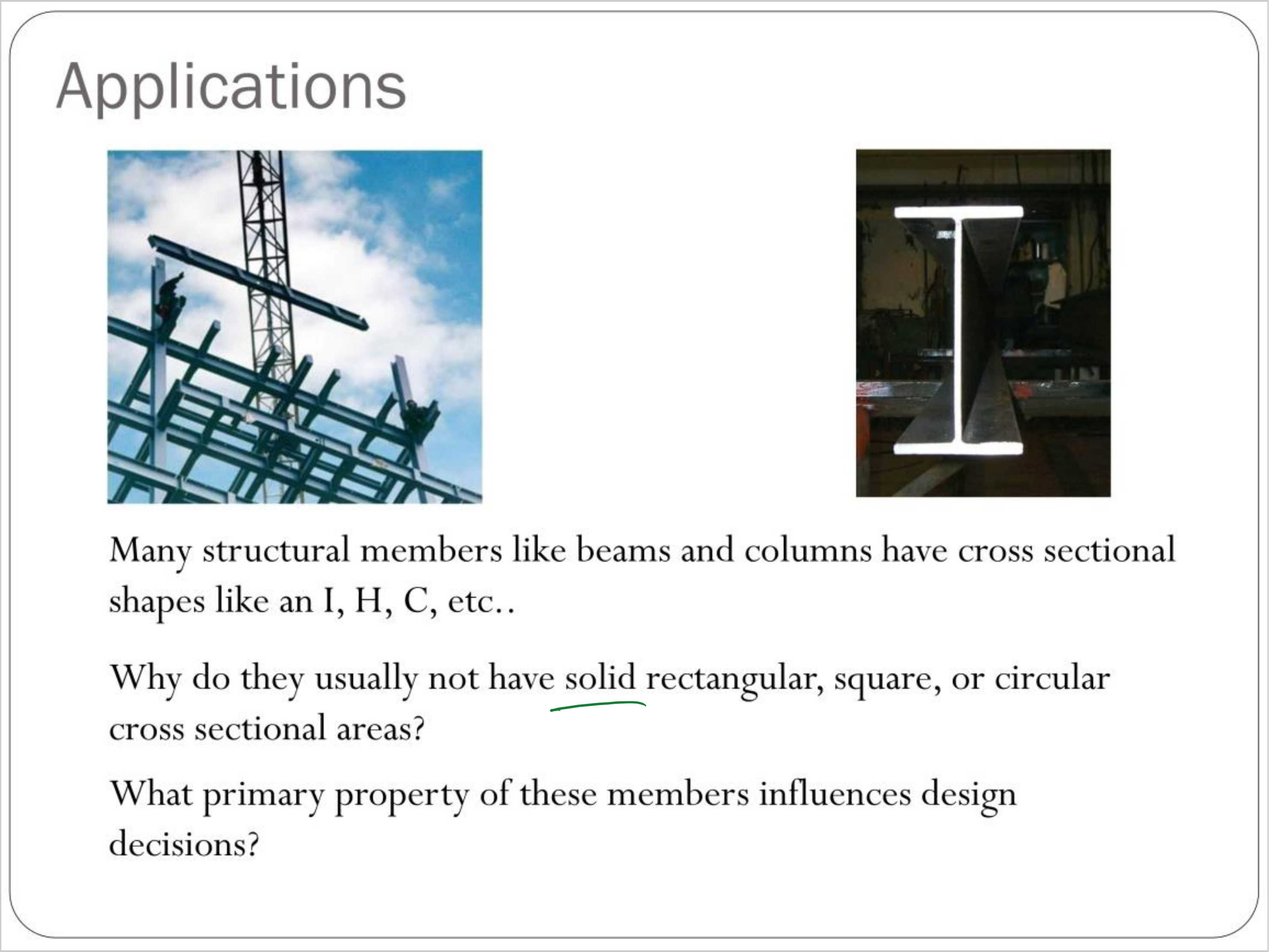

Applications

Many structural members like beams and columns have cross sectional shapes like an I, H, C, etc..

Why do they usually not have solid rectangular, square, or circular cross sectional areas?

What primary property of these members influences design decisions?

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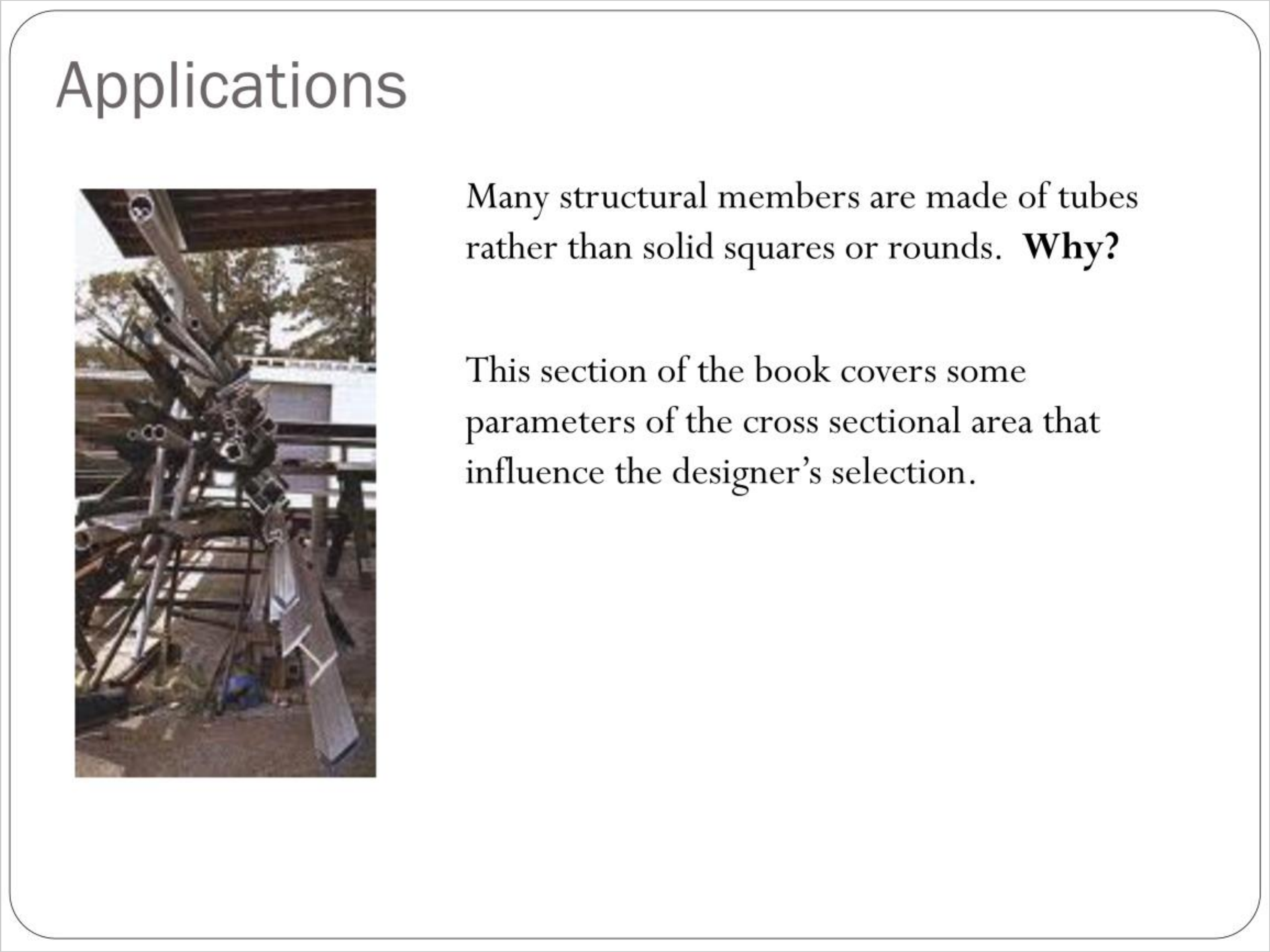
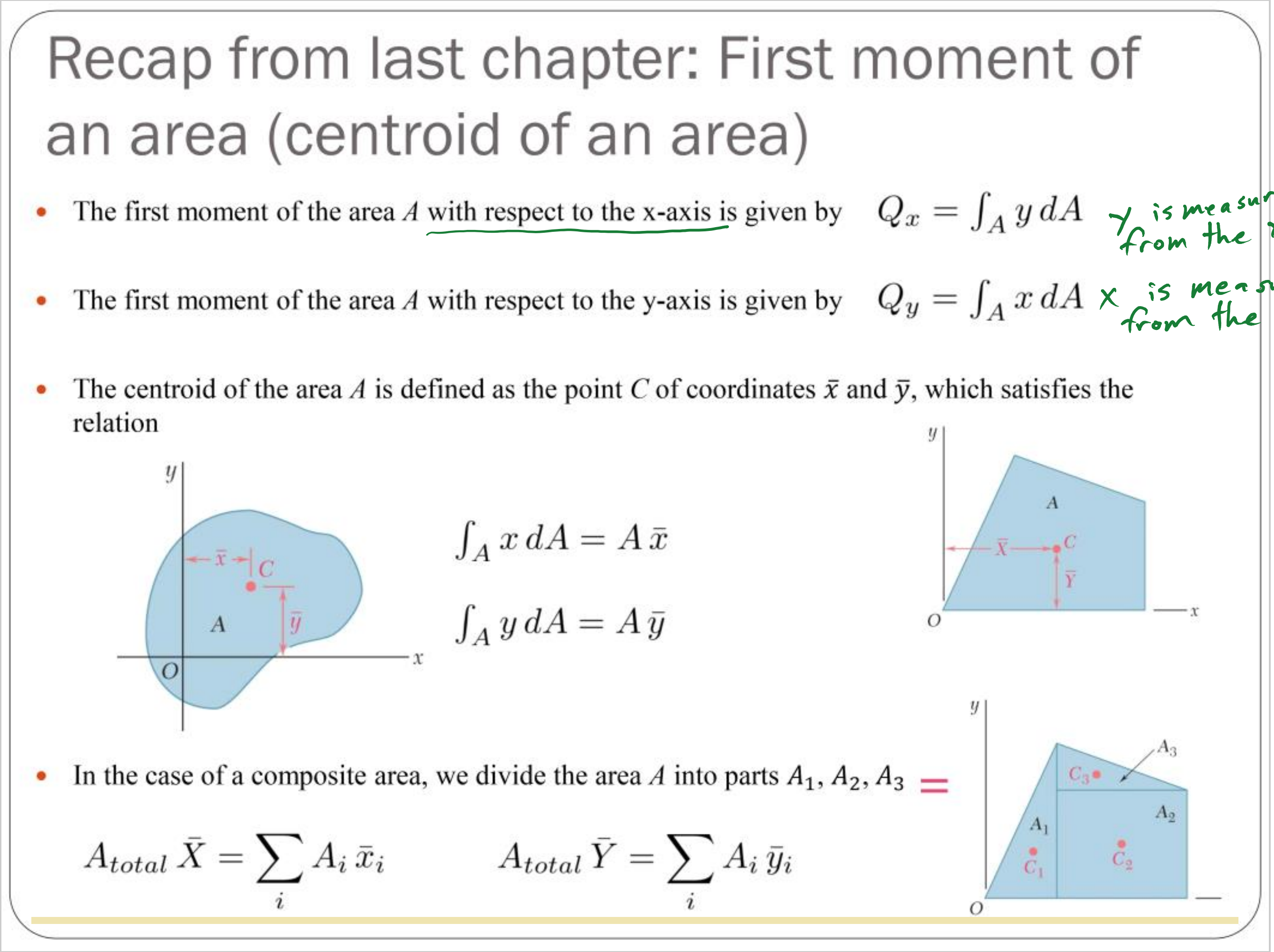


Applications

Many structural members are made of tubes rather than solid squares or rounds. **Why?**

This section of the book covers some parameters of the cross sectional area that influence the designer's selection.



Recap from last chapter: First moment of an area (centroid of an area)

- The first moment of the area A with respect to the x-axis is given by $Q_x = \int_A y\, dA$

y is measured from the x-axis

- The first moment of the area A with respect to the y-axis is given by $Q_y = \int_A x\, dA$

x is measured from the y-axis

- The centroid of the area A is defined as the point C of coordinates $\bar{x}$ and $\bar{y}$, which satisfies the relation

$$\int_A x\, dA = A\,\bar{x}$$

$$\int_A y\, dA = A\,\bar{y}$$

- In the case of a composite area, we divide the area A into parts A_1, A_2, A_3 =

$$A_{total}\,\bar{X} = \sum_i A_i\,\bar{x}_i \qquad A_{total}\,\bar{Y} = \sum_i A_i\,\bar{y}_i$$

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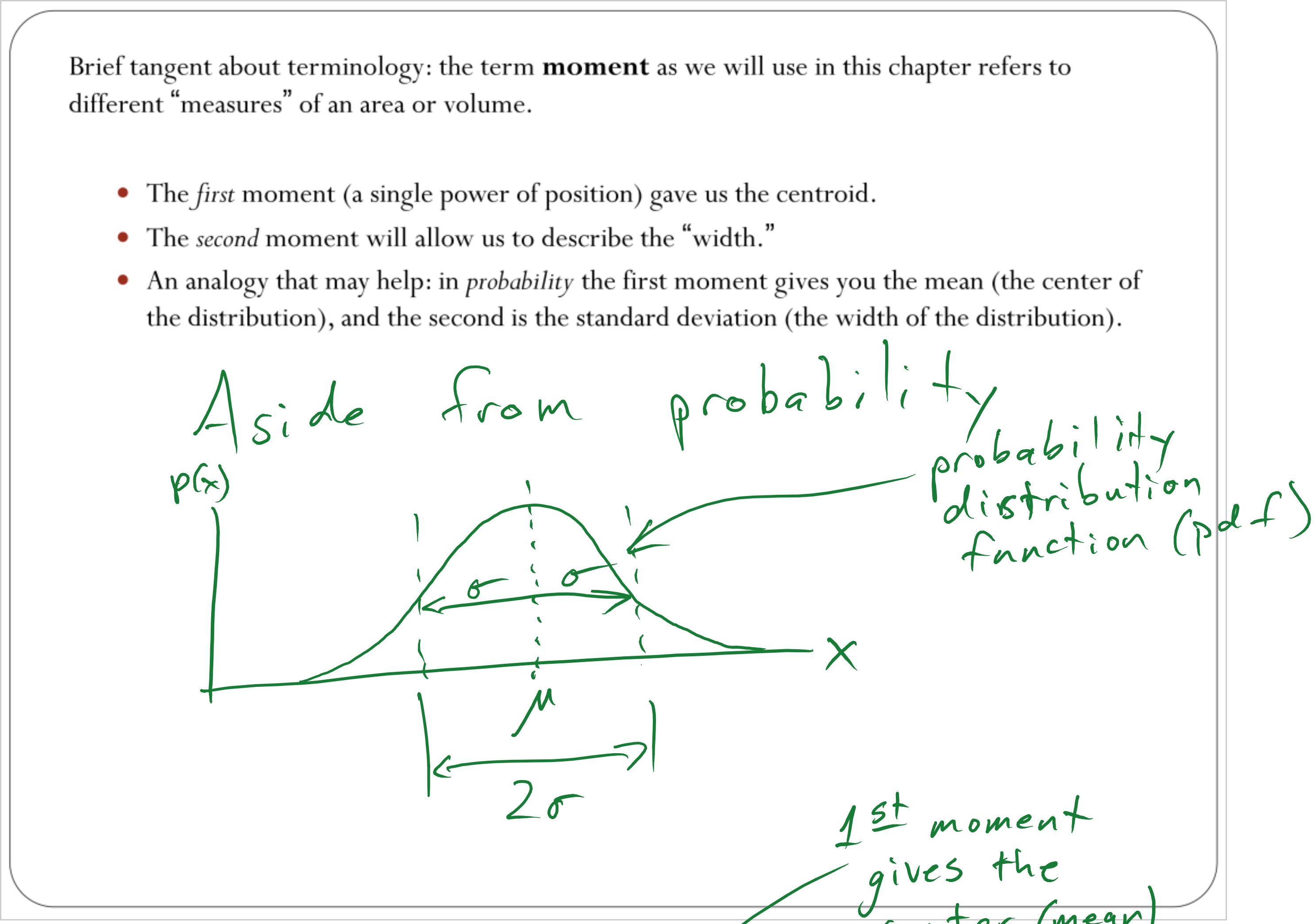

Brief tangent about terminology: the term **moment** as we will use in this chapter refers to different "measures" of an area or volume.

- The *first* moment (a single power of position) gave us the centroid.
- The *second* moment will allow us to describe the "width."
- An analogy that may help: in *probability* the first moment gives you the mean (the center of the distribution), and the second is the standard deviation (the width of the distribution).

Aside from probability

1st moment gives the center (mean) of the pdf

Mean: $\mu = \int x \cdot p(x) \cdot dx$

Variance: $\psi = \int (x-\mu)^2 \cdot p(x) \cdot dx$

2nd moment gives the width of the pdf

standard deviation : $\sigma = \sqrt{\psi}$

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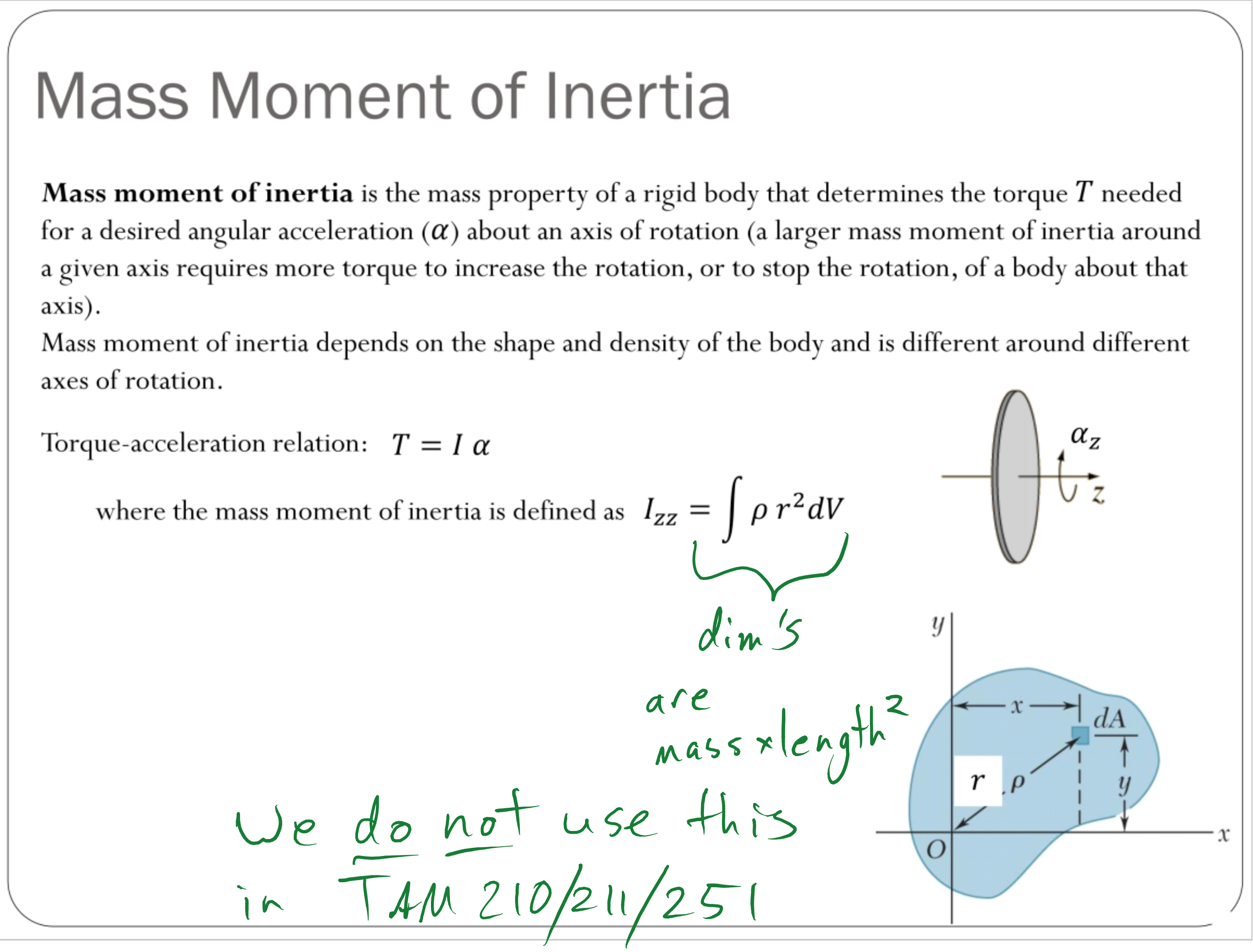


Mass Moment of Inertia

Mass moment of inertia is the mass property of a rigid body that determines the torque T needed for a desired angular acceleration (α) about an axis of rotation (a larger mass moment of inertia around a given axis requires more torque to increase the rotation, or to stop the rotation, of a body about that axis).

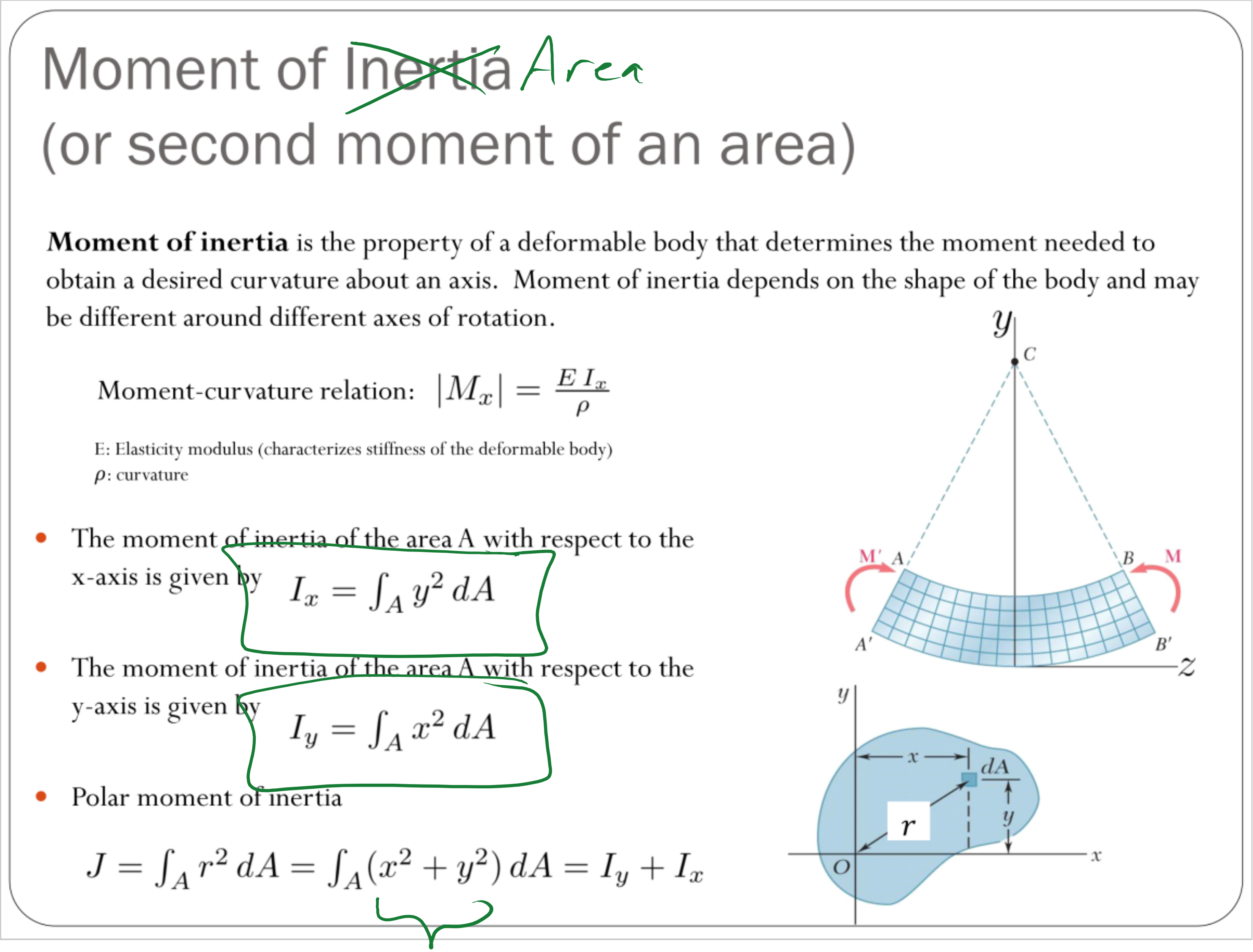
Mass moment of inertia depends on the shape and density of the body and is different around different axes of rotation.

Torque-acceleration relation: $T = I\,\alpha$

where the mass moment of inertia is defined as

$$I_{zz} = \int \underbrace{\rho\, r^2 dV}_{\text{dim's are mass} \times \text{length}^2}$$

We <u>do</u> <u>not</u> use this in TAM 210/211/251



Moment of ~~Inertia~~ Area
(or second moment of an area)

Moment of inertia is the property of a deformable body that determines the moment needed to obtain a desired curvature about an axis. Moment of inertia depends on the shape of the body and may be different around different axes of rotation.

Moment-curvature relation: $|M_x| = \frac{E\,I_x}{\rho}$

E: Elasticity modulus (characterizes stiffness of the deformable body)
ρ: curvature

- The moment of inertia of the area A with respect to the x-axis is given by $\boxed{I_x = \int_A y^2\,dA}$
- The moment of inertia of the area A with respect to the y-axis is given by $\boxed{I_y = \int_A x^2\,dA}$
- Polar moment of inertia

$$J = \int_A r^2\,dA = \int_A \underbrace{(x^2 + y^2)}_{r2}\,dA = I_y + I_x$$

Suppose I_{zz} is the TAM 212 mom. of inertia
which has dim's of $(\text{length})^4$?

A) I_x [✓]
B) I_{zz}
C) B H   In TAM 212,

) I_{zz}

C) Both

D) None

In TAM 212, mom. of inertia has dim's mass·length^2

In TAM 210/211/251

$$I_x = \int_A \underset{\underset{\text{length}^2}{\uparrow}}{y^2} \cdot \underbrace{dA}_{\text{length}^2}$$

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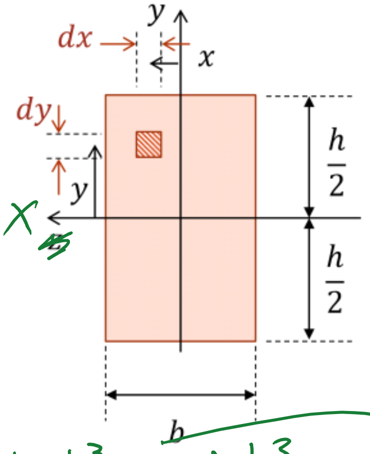
Moment of inertia of a rectangular area

$dA = dx \cdot dy$



$$\boxed{I_x = \int_A y^2 \cdot dA}$$

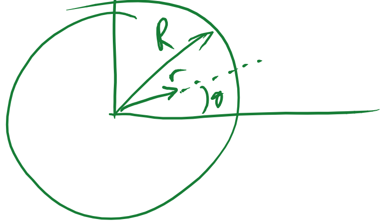
$$= \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} y^2 \cdot dx \cdot dy$$

$$= \int_{-b/2}^{b/2} dx \cdot \int_{-h/2}^{h/2} y^2 dy = b \cdot \int_{-h/2}^{h/2} y^2 dy$$

$$= b\left(\frac{1}{3}y^3\right)\Big|_{-h/2}^{h/2} = \frac{b}{3} \cdot \left[\left(\frac{h}{2}\right)^3 - \left(-\frac{h}{2}\right)^3\right] = \frac{b}{3} \cdot \frac{h^3}{4} = \boxed{\frac{bh^3}{12}}$$

$$I_y = \frac{hb^3}{12}$$

Polar moment of inertia of a circle



$$J = \int_A r^2 \cdot dA = \int_0^{2\pi}\int_0^R r^2 \cdot \underbrace{r \cdot dr \cdot d\theta}_{dA}$$

$$= \int_0^{2\pi} d\theta \cdot \int_0^R r^3 \cdot dr$$

$$= 2\pi \cdot \left(\frac{1}{4}r^4\right)\Big|_0^R$$

$$= \frac{2\pi}{4} R^4 = \frac{\pi R^4}{2}$$

$D = 2R$

$$R = D/2 \Rightarrow J = \frac{\pi \cdot D^4}{32}$$

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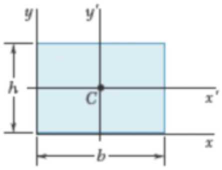
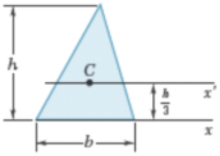
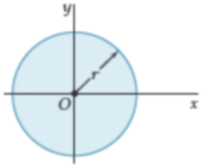
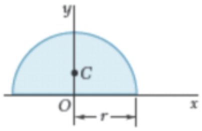
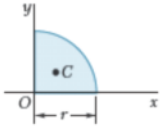
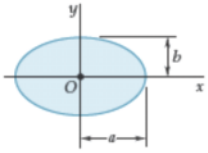
Rectangle		$\overline{I}_{x'} = \frac{1}{12}bh^3$ $\overline{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\overline{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\overline{I}_x = \overline{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Ellipse		$\overline{I}_x = \frac{1}{4}\pi ab^3$ $\overline{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

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Parallel axis theorem

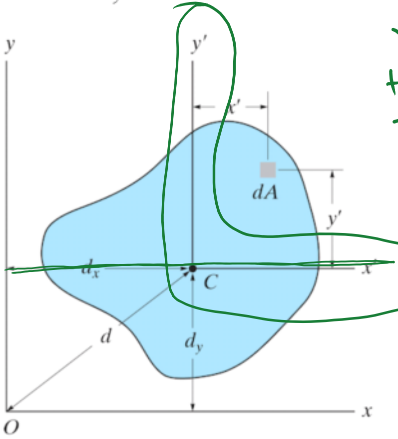
- Often, the **moment of inertia** of an area is known for an axis passing through the **centroid**; e.g., *x'* and *y'*:
- The moments around other axes can be computed from the known $I_{x'}$ and $I_{y'}$:

$$I_x = \int_A y^2 \cdot dA \qquad y = y' + d_y$$

no prime!

$$= \int_A (y' + d_y)^2 dA$$

$$= \int_A \left((y')^2 + 2y' \cdot d_y + d_y^2\right) \cdot dA$$



x', y' pass through the centroid of the area

x, y do not, but they are offset from the centroid by distances d_x & d_y, respectively

Note: the integral over y' gives zero *when done through the centroid axis*.

$$= \underbrace{\int_A (y')^2 dA}_{①} + \underbrace{2\int_A y' \cdot d_y \cdot dA}_{②} + \underbrace{\int_A d_y^2 \cdot dA}_{③}$$

1: $\int_A (y')^2 dA = I_{x'}$

2: $2\int y' \cdot d_y \cdot dA = 2 \cdot \underbrace{d_y}_{} \cdot \int_A y' \cdot dA$ $\qquad$ d_y is constant $\qquad$ $\int y' \cdot dA = 0$

$= 0$

3: $\int_A d_y^2 \cdot dA = d_y^2 \cdot \int_A dA = d_y^2 \cdot A$

$$\Rightarrow \boxed{I_x = I_{x'} + d_y^2 \cdot A}$$ Parallel axis theorem

$$\Rightarrow \boxed{I_x = I_{x'} + d_y^2 \cdot A} \Big\} \text{ theore}$$